## 1) Gamma

$$\theta \sim \text{Gamma}(\alpha,\beta),\quad \alpha > 0,\ \beta > 0$$

$$p(\theta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\theta\beta},\ \theta > 0$$

$$E(\theta) = \frac{\alpha}{\beta}$$

$$V(\theta) = \frac{\alpha}{\beta^2}$$

$$\text{mode}(\theta) = \frac{\alpha-1}{\beta},\quad \alpha \geq 1$$

## 2) Inverse Gamma

$$\theta \sim \text{Inv}-\text{Gamma}(\alpha,\beta),\quad \alpha > 0,\ \beta > 0 \quad \Rightarrow \quad 1/\theta \sim \text{Gamma}(\alpha,\beta)$$

$$p(\theta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{-(\alpha+1)}e^{-\beta/\theta},\ \theta > 0$$

$$E(\theta) = \frac{\beta}{\alpha-1},\quad \alpha > 1$$

$$V(\theta) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)},\quad \alpha > 2$$

$$\text{mode}(\theta) = \frac{\beta}{\alpha+1}$$

## 3) Chi-Square

$$\theta \sim X^2_{\nu},\ \nu > 0 \quad \Rightarrow \quad \theta \sim \text{Gamma}(\alpha = \frac{\nu}{2}, \beta = \frac{1}{2})$$

$$p(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)}\theta^{\nu/2-1}e^{-\theta/2},\ \theta > 0$$

$$E(\theta) = \nu$$

$$V(\theta) = 2\nu$$

$$\text{mode}(\theta) = \nu - 2,\ \nu \geq 2$$

4) **Inverse Chi-Square**

$$\theta \sim \mathrm{Inv}-X^2_\nu,\ \nu > 0 \quad \Rightarrow \quad \theta \sim \mathrm{Inv}-\mathrm{Gamma}(\alpha = \frac{\nu}{2}, \beta = \frac{1}{2})$$

$$p(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} \theta^{-(\nu/2+1)} e^{-1/2\theta},\ \theta > 0$$

$$E(\theta) = \frac{1}{\nu - 2},\ \nu > 2$$

$$V(\theta) = \frac{2}{(\nu-2)^2(\nu-4)},\ \nu > 4$$

$$\mathrm{mode}(\theta) = \frac{1}{\nu+2}$$

5) **Scaled - Inverse Chi-Square**

$$\theta \sim \mathrm{Inv}-X^2(\nu, s^2),\ \nu > 0 \text{ (degrees of freedom)},\ \ s^2 > 0 \text{ (scale)}$$

$$\Rightarrow \quad \theta \sim \mathrm{Inv}-\mathrm{Gamma}(\alpha = \frac{\nu}{2}, \beta = \frac{\nu s^2}{2})$$

$$\text{If } X \sim X^2_\nu \Rightarrow \theta = \frac{\nu s^2}{X} \sim \mathrm{Inv}-X^2(\nu, s^2)$$

$$p(\theta) = \frac{(\nu/2)^{\nu/2}}{\Gamma(\nu/2)} \left(s^2\right)^{\nu/2} \theta^{-(\nu/2+1)} e^{-\nu s^2/2\theta},\ \theta > 0$$

$$E(\theta) = \frac{\nu s^2}{\nu - 2},\ \nu > 2$$

$$V(\theta) = \frac{2\nu^2 s^4}{(\nu-2)^2(\nu-4)},\ \nu > 4$$

$$\mathrm{mode}(\theta) = \frac{\nu s^2}{\nu+2}$$

5) **Scaled - t - distribution**

$\theta \sim t_\nu(\mu, \sigma^2)$, $\nu > 0$ (degrees of freedom), $\mu \in \mathbb{R}$ (location), $\sigma > 0$ (scale)

If $\mu = 0$ and $\sigma = 1$ we have the usual Student distribution $(t_\nu)$ with ν degrees of freedom.

If $X \sim t_\nu \Rightarrow \theta = \mu + \sigma X \sim t_\nu(\mu, \sigma^2)$.

$$p(\theta) = \frac{\Gamma\left[\frac{\nu+1}{2}\right]}{\sigma\sqrt{\nu\pi}\Gamma(\nu/2)}\left[1 + \frac{1}{\nu}\left(\frac{\theta-\mu}{\sigma}\right)^2\right], \theta \in \mathbb{R}$$

$E(\theta) = \mu,\ \nu > 1$

$V(\theta) = \frac{\sigma^2\nu}{(\nu-2)},\ \nu > 2$

$\mathrm{mode}(\theta) = \mu$